For the recursive form of this filter, where each datum is weighted by its position in the chronological sequence, the recursive filter factor for the n^{th} point is given by

$$a_n = \frac{n}{\sum_{i=1}^{n} i} = \frac{2n}{n(n+1)} = \frac{2}{n+1} \qquad (16)$$

Using Eq. (12),

$$\begin{array}{l|l|l} n=1 & a_1 = 1 & \overline{X}_1 = x_1 = 6 \\ \hline n=2 & a_2 = \frac{2}{3} & \overline{X}_2 = 6 + \frac{2}{3}(5-6) = 5.33 \\ \hline n=3 & a_3 = \frac{1}{2} & \overline{X}_3 = 5.33 + \frac{1}{2}(7-5.33) = 6.17 \end{array} \qquad (17)$$

The "memory" (i.e., importance) of older data in this filter fades at a rate dictated by the filter. In this case, the 50^{th} value is 50 times more important than the first, and the 100^{th} value is twice as important as the 50^{th} and 100 times more important than the first.

The exponentially-weighted filter provides the analyst with more flexibility. This filter uses F^i as a weighting factor, where the filter-control constant F is a value chosen between zero and one, and i is the "age-count" of the i^{th} data point. For this filter, $i = 0$ now designates the current or latest data point, $i = 1$ designates the immediately preceding or next-to-last data point, etc., so the data points are indexed in reverse chronological order starting with zero. The weighted least-squares solution is

$$\overline{X}_n = \frac{\sum_{i=0}^{n-1} F^i x_{n-i}}{\sum_{i=0}^{n-1} F^i} \qquad (18)$$

Using F = 0.9 and the same example as before,

$$\begin{aligned} \overline{X}_3 &= \frac{F^0 x_3 + F^1 x_2 + F^2 x_1}{F^0 + F^1 + F^2} \\ &= \frac{(.9)^0(7) + (.9)^1(5) + (.9)^2(6)}{(.9)^0 + (.9)^1 + (.9)^2} \\ &= \frac{7 + 4.5 + 4.86}{2.71} = \frac{16.36}{2.71} = 6.04 \end{aligned} \qquad (19)$$

The weighting of each data point for sample sizes up to 300 is shown in Figure 35 for values of F from 0.8 to 1.0. For F = 1, all points in the sample are weighted equally. For