$$\begin{aligned} \bar{X}_n &= \bar{X}_{n-1}(1-a_n) + x_n(a_n) \\ \bar{X}_n &= \bar{X}_{n-1} + a_n\left(x_n - \bar{X}_{n-1}\right) \end{aligned} \tag{12}$$

For the equally-weighted case, the recursive filter factor $a_n = 1/n$.

Using the same example, with $\bar{X}_o = 0$,

$$\begin{aligned} \bar{X}_1 &= x_1 = 6 \\ \bar{X}_2 &= \bar{X}_1 + \frac{1}{2}\left(x_2 - \bar{X}_1\right) = 6 + \frac{1}{2}(5-6) = 5.5 \\ \bar{X}_3 &= \bar{X}_2 + \frac{1}{3}\left(x_3 - \bar{X}_2\right) = 5.5 + \frac{1}{3}(7-5.5) = 6.0 \end{aligned} \tag{13}$$

In general terms, this recursive formulation of the least squares solution is called an expanding-memory filter, as opposed to a sliding-window or fixed-length filter. In an expanding-memory filter, the solution is always based on the entire data set. In the equally-weighted case, all data points have an equal influence on the solution, regardless of their locations in the sequence.

It can be seen that in the limit as n becomes very large, a_n approaches zero. That is, each data point in the sequence is accorded a decreased weight due to the increased number of points being fit. If the data being fit should actually describe a constant, this is exactly what is desired. Normally, however, the function that the data should fit is unknown, and a constant function is used merely as an approximation to smooth or edit the data. What is desired is a recursive least squares fit that assigns a decreasing weight to data of increasing age, so the fit de-weights data points used in earlier recursions.

In a fading-memory filter, the weighting factor decreases as time recedes into the past, so that the importance of any given datum will decrease as the age of the datum increases. An example of such a filter is one in which each datum is weighted by its count or index number in the sequence:

$$\bar{X}_n = \frac{\sum_{i=1}^{n} i\, x_i}{\sum_{i=1}^{n} i} \tag{14}$$

Using the same numerical example as before, where $x_1 = 6$, $x_2 = 5$, and $x_3 = 7$,

$$\bar{X} = \frac{1\cdot 6 + 2\cdot 5 + 3\cdot 7}{1+2+3} = \frac{37}{6} = 6.17 \tag{15}$$