Appendix B. Shaping-Constant Effects on Mode-5 Impact Distributions

The values chosen for shaping constants A and B that appear in the Mode-5 impact-density function [Eq. (3)] have a significant effect on the angular distribution of impacts about the launch point. This Appendix shows the effects of A and B on (1) the ratio of impacts along the downrange line to any other radial through the launch point, and (2) the percentages of impacts in various sectors relative to the downrange line.

Following the procedures outlined in Section 9.7 of Reference [1], it is interesting to observe the effects of varying the constants A and B. This is done in terms of a so-called f-ratio, which is expressed in Ref. [1] as Eq. (9.19), and is repeated here:

$$f-\text{ratio} = \frac{e^{A\pi} + \frac{B}{R}}{e^{A\phi} + \frac{B}{R}} \tag{7}$$

The ratio shows how much more likely impact is to occur along the flight line (where $\phi = \pi$) than along some other radial line that makes an angle θ ($\theta = \pi - \varphi$) with the flight line. Table 33 and Table 34 present f-ratios for values of $A = 2.5$, 3.0, 3.5, and 4.0, and $B = 1000$ for impact ranges from one to 25 miles. Table 35 and Table 36 show the effects of halving and doubling the constant B for a fixed value of $A = 3.0$.

Before citing numerical examples, it should be emphasized that the data in Table 33 through Table 36 are derived from the primary Mode-5 impact-density function and, as such, they indicate likelihood ratios for the location of the secondary Mode-5 density functions. A secondary function, it will be remembered, describes the dispersion of a debris class about the impact point of the mean piece in the class. Thus, referring to Table 34 with $A = 3.0$, it can be seen that the secondary impact-density function for a debris class is 4.7 times more likely to be centered 10 miles downrange along the flight line ($\theta = 0°$) than 10 miles from the launch point along a radial line that makes a 30° angle with the flight line. As another example, the secondary function (i.e., the impact point for the mean piece in a debris class) is 82.2 times more likely to be located 25 miles downrange along the flight line than 25 miles crossrange ($\theta = 90°$), and assuming no destruct action, that it is $303.2/82.2 = 3.7$ times more likely to be located 25 miles crossrange than 25 miles uprange ($\theta = 180°$).